Thermal Noise Propagation in Water-fat Imaging and Fat Fraction Measurement

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TARGET AUDIENCE: Radiologists and MR physicists who are interested in quantitative water-fat imaging.

PURPOSE: Quantitative measurements in MRI require simultaneous precision and accuracy. When making clinical decisions based on these measurements, it is desirable to know the variability in the presence of thermal noise. Hansen et al. [1] recently proposed a simple and efficient method for determining the variance of certain region of interest (ROI) measurements due to thermal noise. This approach has been extended to phase-contrast MRI [2]. The purpose of this work is to further extend the approach to water-fat imaging and the in-vivo measurement of proton density fat fraction and water fraction. We cross validate the result using the pseudo-replica technique [3].

$$s_{t_n} = \left[\sum_p a_p e^{j2\pi f_p t_n}\right] e^{j2\pi\chi t_n} \qquad \text{(Eqn. 1)}$$

$$\underline{w} = (1, j, 0, 0)^T, \underline{f} = (0, 0, 1, j)^T \qquad \text{(Eqn. 2)}$$

$$\begin{aligned} \sigma^2_{w,m} &= \underline{w}^H A^\dagger E_\chi \Sigma_{s'_m} E^H_\chi \left(A^\dagger\right)^H \underline{w} \\ \sigma^2_{f,m} &= \underline{f}^H A^\dagger E_\chi \Sigma_{s'_m} E^H_\chi \left(A^\dagger\right)^H \underline{f} \end{aligned} \qquad \text{(Eqn. 3)}$$

METHODs: *Algorithm:* The noise-free water-fat signal is modeled using **Eqn. 1**, where s_{tn} is the signal at each echo time t_n; a_p and f_p are the relative amplitudes and frequencies in the multi-peak fat model [4], respectively; $\chi=\varphi+jR_2^*$ is the estimated complex field map in Hz. Signal compensation can be written as $s_m'=E_\chi s_m$, where E_χ is a diagonal matrix with the n^th^ and the 2n^th^ diagonal elements being real and imaginary part of $\exp(-j2\pi\chi t_n)$. Assuming that we have prior knowledge or accurate estimates of the field heterogeneity (φ and R_2^*), the signal after compensation can be written in matrix form as $s_m'=A\rho_m$, where m denotes the m^th^ pixel, and vector s_m', ρ_m and matrix A follow the same form as Appendix A from Ref [5]. The covariance matrix of the linear transform Ax is $A\Sigma_x A^H$, where Σ_x is the covariance matrix of x. We obtain an expression for the covariance matrix of ρ_m by concatenating the corresponding linear operators. The 2N-by-2N covariance matrix Σ_{sm} of signal vector s_m is obtained through the process described in Ref [1]. Defining vectors w and f as **Eqn. 2**, we obtain the variances of water and fat images for each pixel using **Eqn. 3**. Because fat and water fraction (e.g. w/(w+f)) involves a nonlinear operation, the variance of fat fraction measurements cannot be obtained in the same way. We use an approximation based on Taylor-series [6] to achieve a closed form solution. *Experimental Methods:* The flowchart describing our validation experiment is shown in **Fig. 1**. In the experiment, we scanned a human volunteer. Fully sampled k-space data were collected using an 8 channel cardiac coil with the 6-echo IDEAL sequence on a GE 3T scanner, TE_1 1.4ms, ΔTE 1.0ms, TR 9ms, flip angle 3°, BW 62.5KHz. Noise samples were collected using the same sequence with RF excitation turned off. Water-fat separation was performed using the ISMRM fat-water toolbox [7] with the graph cut method [8]. Noise propagation and pseudo-replica technique were implemented in MATLAB (Mathworks, Natick, MA) on a Linux workstation (2.93GHz/12-core Intel Xeon X5670 CPU and 128 GB RAM). We simulated 10^4 pseudo-replicas for reference. Three ROIs were manually selected in the liver.

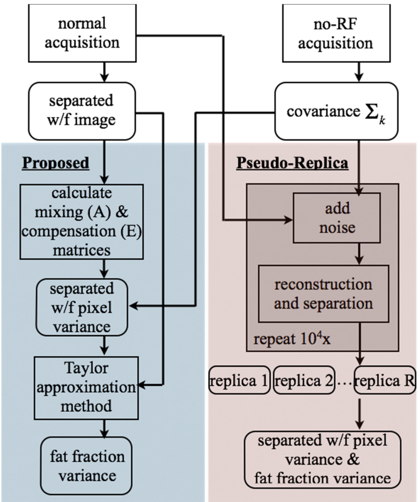

Figure 1: Experiment pipeline: The left branch shows noise propagation calculation; right branch shows pseudo-replica technique for verification purpose.

RESULTS AND DISCUSSION: **Fig. 2** shows (top row) the standard deviation (STD) of the water and fat signals normalized by the total signal |w|+|f|. These maps are in good agreement with (bottom row) ones obtained from pseudo-replica technique. **Table 1** compares the results of quantitative fat fraction measurement in 3 different ROIs shown on the right. Results from proposed method conformed to the ones resulting from pseudo-replica technique. The pseudo-replica technique required roughly 100 minutes, while the proposed method required 1/3 minute for all computation (300x faster).

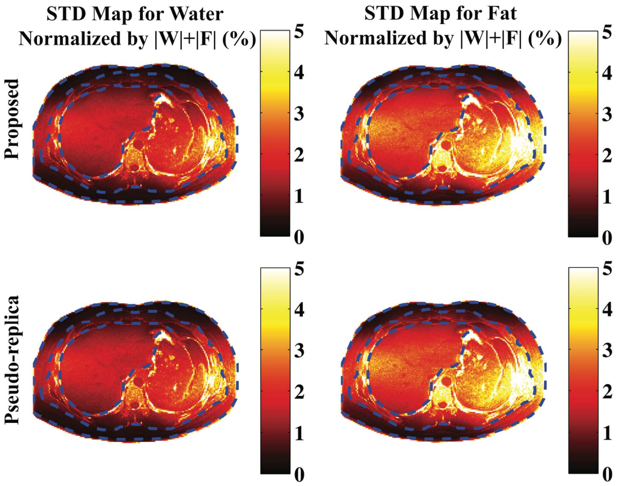

Figure 2: Standard deviation maps as a percentage of |w|+|f| were generated by proposed method and pseudo-replica technique for cross validation. Blue dash indicated subcutaneous fat and liver.

CONCLUSION: We have demonstrated a fast and efficient method to calculate the variability of the estimated water and fat signal and proton density fat fraction in ROIs due to thermal noise. This method has been cross validated against the pseudo-replica technique.

REFERENCE: [1] Hansen MS et al, MRM 2014 (early view). [2] Hansen MS et al, JCMR 2014 16:46. [3] Robson PM et al, MRM 2008 60:895-907. [4] Brix G et al. MRI 1993 11:977-91. [5]. Reeder SB et al, MRM 2004 51:35-45. [6]. Casella G et al. Statistical Inference: 240-245. [7]. Hu HH et al, MRM 2012 68:378-88. [8]. Hernando D et al, MRM 2010 63:79-90.

	ROI 1	ROI 2	ROI 3
Proposed Method	0.144%	0.242%	0.182%
Pseudo-replica	0.145%	0.274%	0.180%

Table 1: Standard deviation of liver ROI fat fraction measurements from one subject.